Lecture 27 Summary

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1 The Kosterlitz-Thouless Phase Transition for 2D Superconductors

Last time we saw that the energy of a V/AV pair is significantly lower than that of an isolated vortex. The energy of a free vortex can be calculated by ignoring the vortex core (GL $\kappa \to \infty$) and considering only the kinetic energy of the currents as,
$W_1 = \pi n^*_{s,2D} \frac{\hbar^2}{m^*} \ln \frac{R}{r_0}$, where $n^*_{s,2D} = n_s L$ is the 2D superfluid density, n_s is the 3D superfluid density, L is the length of the vortex (on the order of the film thickness), r_0 is the microscopic length scale where the current density approaches the de-pairing value (we expect $r_0 \sim \xi_{GL}$), and R is the sample size, where it is assumed that $\lambda_\perp$ is much greater than the sample size. The energy of a single isolated vortex scales with the system size, making it very expensive!

Contrast this with the case of a V/AV pair at some distance r apart. Far away ($R >> r$) the flow fields of the two vortices cancel to good approximation, making the object appear "neutral" from far away. The currents are strong only within r, giving rise to a total energy of just,
$W_2 = 2\pi n^*_{s,2D} \frac{\hbar^2}{m^*} \ln \frac{r}{r_0}$.
Because $W_2 << W_1$ the V/AV excitations are the dominant excitations at low temperature in the 2D superconductor.

To naively estimate the KT transition temperature T_{KT} calculate the Helmholtz free energy of a free vortex, ΔF_1 and see where it changes sign. The entropy comes from counting the number of microscopic configurations that give the same macroscopic properties. In the case of a free vortex added to the sample, the vortex could be located in any square of size a, where a is expected to be on the order of r_0. Thus the Helmholtz free energy can be written as,
$\Delta F_1 = W_1 - k_B T \ln(R^2/r_0^2)$.
This can be expanded as,
$\Delta F_1 = \left(\pi n^*_{s,2D} \frac{\hbar^2}{m^*} - 2k_B T\right) \ln \frac{R}{r_0} - 2k_B T \ln \frac{r_0}{a}$.
In the thermodynamic limit $R \to \infty$ only the first term survives.

Looking at the temperature where $\Delta F_1 = 0$ yields this implicit equation for T_{KT}:
$n^*_{s,2D}(T_{KT}) = \frac{2m^* k_B}{\pi \hbar^2} T_{KT}$. One can find T_{KT} by finding the intersection of $n^*_{s,2D}(T)$ and the line described by $\frac{2m^* k_B}{\pi \hbar^2} T$. The class web site shows such data from superfluid ^4He and In/InOx superconducting films.

2 Highlights of KT Physics in 2D Superconductors

For temperatures above T_{KT} one can define a free-vortex correlation length $\xi_+(T) \sim r_0 e^{\sqrt{B \frac{T_{KT}}{T - T_{KT}}}}$, where B is a constant of order unity. This is a measure of the puddle size of free-vortex-free regions. Note that this length scale diverges as T_{KT} is approached from above. One can use it to estimate the free vortex density as $n_f(T) = 1/\xi_+^2(T)$, for $T > T_{KT}$. The free vortex density thus goes to zero at T_{KT}. The free vortices will dissipate energy when acted upon by an external current, thus T_{KT} can be found from the zero-resistance state of the material, in principle.

When a transport current is applied to a bound V/AV pair, the Lorentz force will act in opposite directions on each vortex and act to stretch the pair. This gives rise to a peak in the energy of the V/AV pair as a function of separation r. The V/AV pair can unbind due to a thermal fluctuation activating the system over the barrier, creating free vortices below T_{KT}. The free-vortex generation rate is given by
$G = G_0 e^{-E_0/k_B T}$, where $E_0 = q^2 \ln\left(\frac{q^2}{r_0 \Phi_0 j_{2D}}\right)$, G_0 is the attempt frequency, $q^2(T) = \frac{2\pi \hbar^2 n^*_{s,2D}(T)}{m^*}$, and j_{2D} is the 2D surface current density. With these definitions, the free vortex generation rate is,
$G = G_0 \left(\frac{r_0 \Phi_0 j_{2D}}{q^2}\right)^{q^2/k_B T}$.
But free vortices also can re-combine and annihilate. This recombination rate is given by $R = R_0 n_f^2$, where n_f is the free vortex density.
By assuming equilibrum and equating the generation and recombination rates, we can calcualte the free vortex density as,
$n_f = \sqrt{\frac{G_0}{R_0}} \left(\frac{r_0 \Phi_0 j_{2D}}{q^2}\right)^{q^2/2k_B T}$, for $T < T_{KT}$ in the presence of a current.

These free vortices will create a longitudinal electric field given by,
$E \sim j_{2D}^{a(T)}$ with $a(T) = 1 + \frac{\pi \hbar^2 n^*_{s,2D}(T)}{m^* k_B T}$. This exponent has the value of 3 at T_{KT}, and a value of 1 above T_{KT} (Ohmic dissipation due to free vortices). More generally, the $E - j_{2D}$ relation can be written as $E \sim j_{2D}^{1+2\frac{T_{KT}}{T}\frac{n^*_{s,2D}(T)}{n^*_{s,2D}(T_{KT})}}$. This form shows that the exponent grows from a value of 3 for $T < T_{KT}$. Thus the

IV curves show a discontinuous jump in slope from 1 to 3 at T_{KT}, followed by a steady rise below that temperature. The large value of the exponent at low temperature resembles a finite critical current.

Along with this there is a discontinuous drop to zero in superfluid density $n^*_{s,2D}$ at T_{KT}.